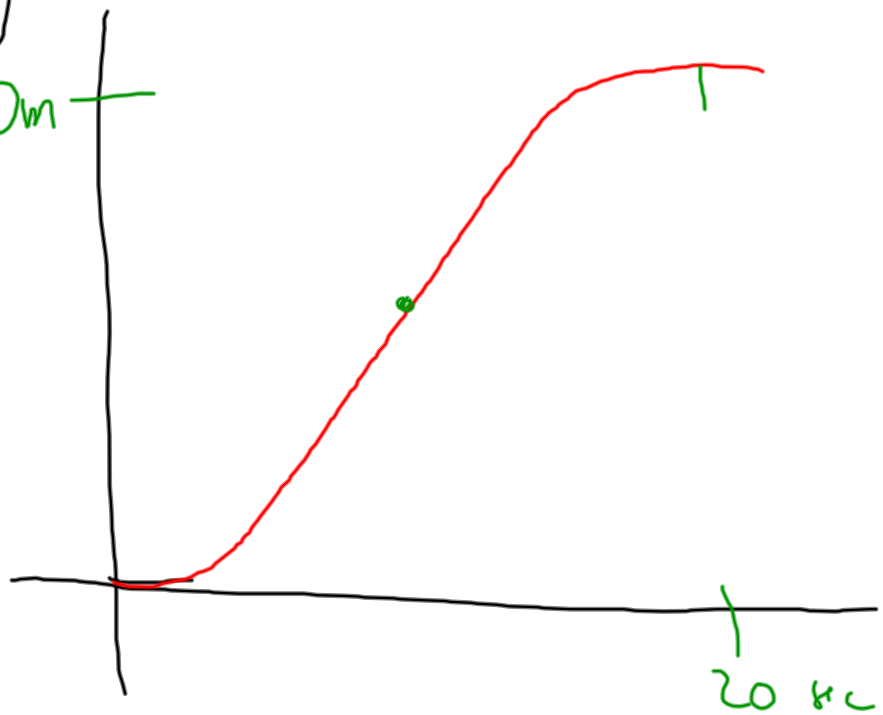
3.1 ①



Alg 2:

Speed = $\frac{\Delta \text{ distance}}{\Delta \text{ time}}$

"rate"
"velocity"

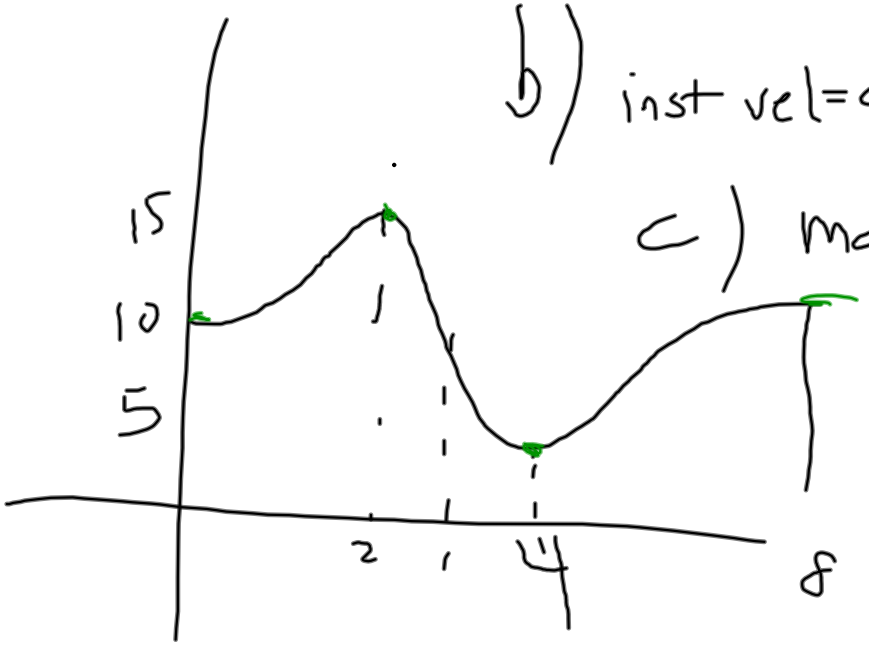
= "avg velocity"

instantaneous velocity =

velocity at an instant

= estimate w/ slope of a line

2) a) $\frac{pos(3)-pos(0)}{3-0} = \frac{10-0}{3-0} = \frac{0}{3} = 0$



b) inst vel = 0

c) max (min) inst vel

d) inst vel @ t = 3

-Scm/sec ★

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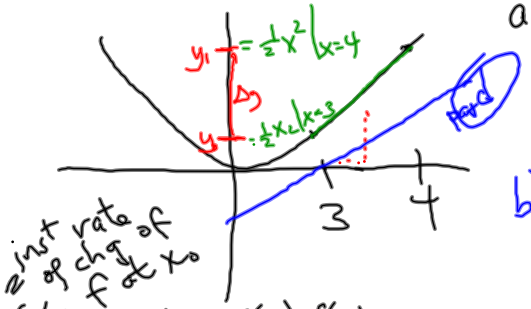
#7) $y = \frac{1}{2}x^2$; $x_0 = 3$; $x_1 = 4$

$d = rt$, $\frac{d}{t} = r$



inst rate of chg of f at x_0

$$f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

a) Avg rate of chg $= \frac{\text{total chg } y}{\text{total chg } x} = \frac{\left(\frac{1}{2}4^2\right) - \left(\frac{1}{2}3^2\right)}{4-3}$

$= \frac{8 - \frac{9}{2}}{1} = \frac{7}{2}$

b) inst rate of chg $= f'(x_0) = \lim_{x_1 \to x_0} \frac{\frac{1}{2}x_1^2 - \frac{1}{2}x_0^2}{x_1 - x_0}$

$\Rightarrow f'(3) = \lim_{x_1 \to 3} \frac{\frac{1}{2}x_1^2 - \frac{9}{2}}{x_1 - 3}$

$= \frac{1}{2}\lim_{x_1 \to 3} \frac{x_1^2 - 9}{x_1 - 3} = \frac{1}{2}\lim_{x_1 \to 3} \frac{(x_1 - 3)(x_1 +$

$f'(3) = \frac{1}{2}\lim_{x_1 \to 3}(x_1 + 3) = \frac{1}{2}(3+3) =$

inst rate of chg $= 3$

c) $f'(x_0) = \lim_{x_1 \to x_0} \frac{\frac{x_1^2}{2} - \frac{x_0^2}{2}}{x_1 - x_0}$

$= \frac{1}{2}\lim_{x_1 \to x_0} \frac{x_1^2 - x_0^2}{x_1 - x_0}$

$= \frac{1}{2}\lim_{x_1 \to x_0} \frac{(x_1 - x_0)(x_1 + x_0)}{(x_1 - x_0)} = \frac{1}{2}\lim_{x_1 \to x_0}(x_1 + x_0) = \frac{1}{2}(x_0 + x_0)$

$= \frac{1}{2}(2x_0) = x_0$

d)

8) $y = x^3$; $x_0 = 1$; $x_1 = 2$

$f(x_0) = (1)^3 = 1$

$f(x_1) = (2)^3 = 8$

a) avg r.o.c. $= \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{8-1}{2-1} = \dfrac{7}{1} = 7$

b) inst r.o.c. $= f'(1) = \lim\limits_{x \to 1} \dfrac{f(x_1) - f(1)}{x_1 - 1} = \lim\limits_{x_1 \to 1} \dfrac{x_1^3 - 1}{x_1 - 1}$

$= \lim\limits_{x_1 \to 1} \dfrac{(x_1 - 1)(x_1^2 + (1)x_1 + (1)^2)}{(x_1 - 1)}$

$= \lim\limits_{x_1 \to 1} x_1^2 + x_1 + 1 = 1 + 1 + 1 = 3$

c) inst r.o.c. $=$

$f'(x_0) =$

$\lim\limits_{x_1 \to x_0} \dfrac{x_1^3 - x_0^3}{x_1 - x_0}$

$= \lim\limits_{x_1 \to x_0} \dfrac{(x_1 - x_0)(x_1^2 + x_0 x_1 + x_0^2)}{(x_1 - x_0)} = \lim\limits_{x_1 \to x_0} x_1^2 + x_0 x_1 + x_0^2 = x_0^2 + x_0 \cdot x_0 + x_0^2$

$= 3x_0^2$

d)

11) $f(x) = x^2 + 1$; $x_0 = 2$

a) $f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{(x_1^2 + 1) - (x_0^2 + 1)}{x_1 - x_0}$

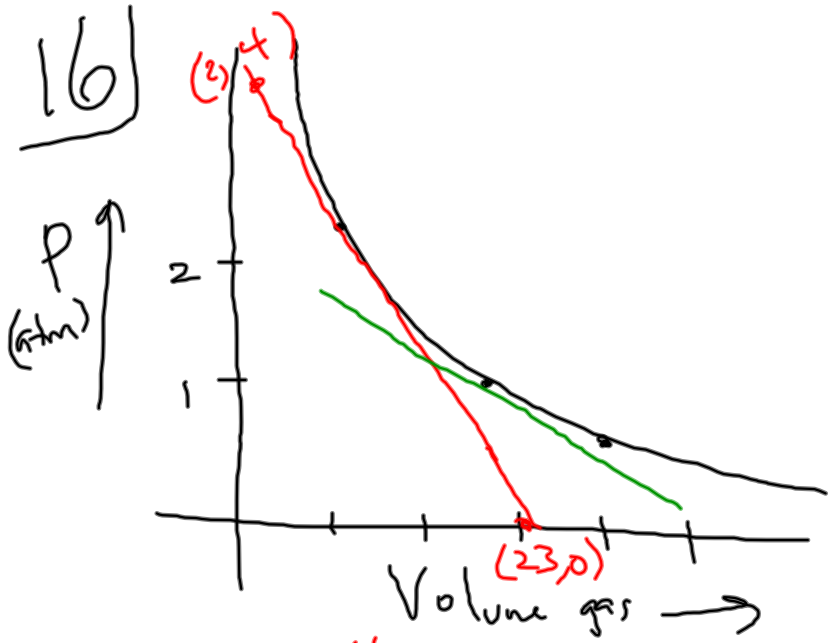
$= \lim_{x_1 \to x_0} \frac{x_1^2 - x_0^2}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{(x_1 - x_0)(x_1 + x_0)}{(x_1 - x_0)}$

$= \lim_{x_1 \to x_0} (x_1 + x_0) = x_0 + x_0 = 2x_0$

b) find inst rate of chg @ x_0 ...

$f'(x_0) = 2x_0$ then $f'(2) = 2(2) = 4$

16)



$$\text{est} = \frac{0-4}{23-2} = \frac{-4}{21} \approx -.19$$

Estimate instantaneous rate of chg by slope of tangent line

12) $f(x) = x^2 + 3x + 2$; $x_0 = 2$ {p. 13

a) instroc @ $x = x_0$ $= f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{(x_1^2 + 3x_1 + 2) - (x_0^2 + 3x_0 + 2)}{x_1 - x_0}$

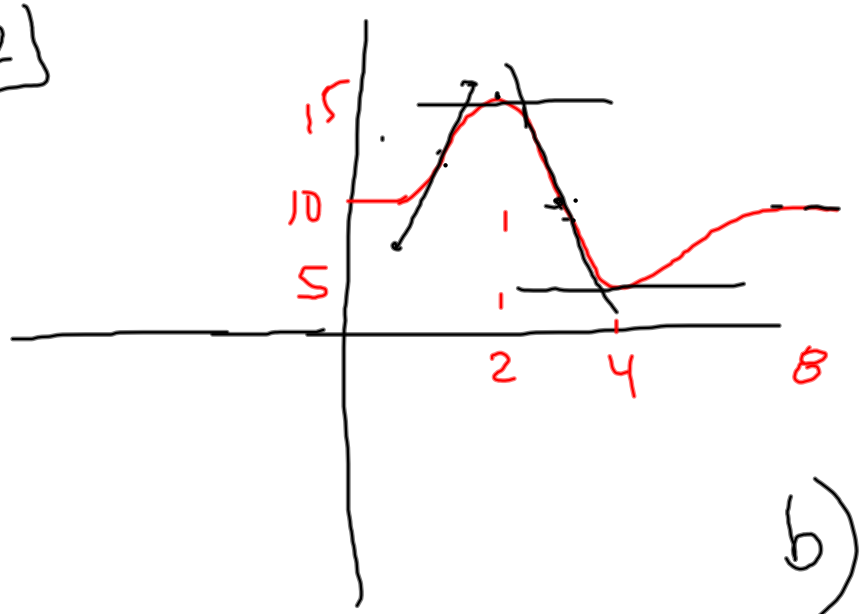
$$= \lim_{x_1 \to x_0} \frac{(x_1^2 - x_0^2) + (3x_1 - 3x_0) + (2 - 2)}{(x_1 - x_0)}$$

$$= \lim_{x_1 \to x_0} \frac{(x_1 - x_0)(x_1 + x_0) + 3(x_1 - x_0)}{(x_1 - x_0)}$$

$$= \lim_{x_1 \to x_0} \frac{(x_1 - x_0)(x_1 + x_0)}{(x_1 - x_0)} + \lim_{x_1 \to x_0} \frac{3(x_1 - x_0)}{(x_1 - x_0)}$$

$$= \lim_{x_1 \to x_0} (x_1 + x_0) + \lim_{x_1 \to x_0} 3 = 2x_0 + 3$$

2)



a) avg vel [0,3]

Avg: "velocity" "speed" "rate"

$$\frac{p(3)-p(0)}{3-0} = \frac{10-10}{3} = 0$$

b) inst vel = 0

c) max (min) velocities

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10) $y = \frac{1}{x^2}$; $x_0 = 1$; $x_1 = 2$

a) avg rate of chg = slope of line seg. joining $(x_1, f(x_1))$ and $(x_0, f(x_0))$

$$= \frac{f(2)-f(1)}{2-1} = \frac{\frac{1}{2^2} - \frac{1}{1^2}}{2-1} = \frac{\frac{1}{4}-1}{1} = \boxed{-\frac{3}{4}}$$

b) inst roc @ $x_0 = 1 \Rightarrow f'(1) = \lim_{x_1 \to 1} \frac{f(x_1)-f(1)}{x_1-1}$

$f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1)-f(x_0)}{x_1-x_0}$ P173

$$= \lim_{x_1 \to 1} \frac{\frac{1}{x_1^2} - \frac{1}{1^2}}{x_1-1} = \lim_{x_1 \to 1} \frac{\frac{1-x_1^2}{x_1^2}}{(x_1-1)} = \lim_{x_1 \to 1} \frac{\frac{(1-x_1)(1+x_1)}{x_1^2}}{(x_1-1)}$$

OK

$$\lim_{x_1 \to 1} \frac{\frac{(1-x_1)(1+x_1)}{x_1^2}\left(\frac{1}{x_1-1}\right)}{(x_1-1)\left(\frac{1}{x_1-1}\right)} = \lim_{x_1 \to 1} \frac{\frac{(1-x_1)}{(x_1-1)}\frac{(1+x_1)}{x_1^2}}{\frac{x_1-1}{x_1-1}}$$

$\frac{a}{\frac{b}{c}} = \frac{a}{bc}$

$$= \lim_{x_1 \to 1} \frac{(-1)\left(\frac{x_1+1}{x_1^2}\right)}{1} = -1\left(\frac{2}{1}\right) = \boxed{-2}$$

c) inst roc @ $x = x_0$

$$\lim_{x_1 \to x_0} \frac{\frac{1}{x_1^2} - \frac{1}{x_0^2}}{x_1-x_0} = \lim_{x_1 \to x_0} \frac{\frac{x_0^2-x_1^2}{x_0^2x_1^2}}{(x_1-x_0)} = \lim_{x_1 \to x_0} \frac{\frac{(x_0-x_1)(x_0+x_1)}{x_0^2x_1^2}}{(x_1-x_0)}$$

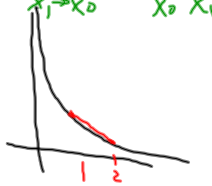
(-1)

$$= \lim_{x_1 \to x_0} \frac{(x_0-x_1)(x_0+x_1)}{x_0^2x_1^2(x_1-x_0)} = \lim_{x_1 \to x_0} (-1)\left(\frac{x_0+x_1}{x_0^2x_1^2}\right) = \frac{-2x_0}{x_0^4}$$

x_0+x_0

$$= \frac{-2}{x_0^3}$$

d)



(4) $\frac{1}{\sqrt{x}}$; $x_0 = 4$

a) $\lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \lim_{x_1 \to x_0} \frac{\frac{1}{\sqrt{x_1}} - \frac{1}{\sqrt{x_0}}}{x_1 - x_0}$

$\frac{\sqrt{x_0}}{\sqrt{x_0}\sqrt{x_1}} - \frac{\sqrt{x_1}}{\sqrt{x_0}\sqrt{x_1}} = \frac{\sqrt{x_0} - \sqrt{x_1}}{\sqrt{x_0}\sqrt{x_1}}$

$= \lim_{x_1 \to x_0} \frac{\left(\frac{\sqrt{x_0} - \sqrt{x_1}}{\sqrt{x_0}\sqrt{x_1}}\right)\left(\frac{\sqrt{x_0} + \sqrt{x_1}}{1}\right)}{(x_1 - x_0)\left(\frac{\sqrt{x_0} + \sqrt{x_1}}{1}\right)}$

$= \lim_{x_1 \to x_0} \frac{x_0 - x_1}{\sqrt{x_0 x_1}(x_1 - x_0)(\sqrt{x_0} + \sqrt{x_1})}$

$= \lim_{x_1 \to x_0} -\frac{1}{\sqrt{x_0 x_1}(\sqrt{x_0} + \sqrt{x_1})}$

$= -\frac{1}{\sqrt{x_0^2}(\sqrt{x_0} + \sqrt{x_0})}$

$= -\frac{1}{|x_0|(2\sqrt{x_0})}$

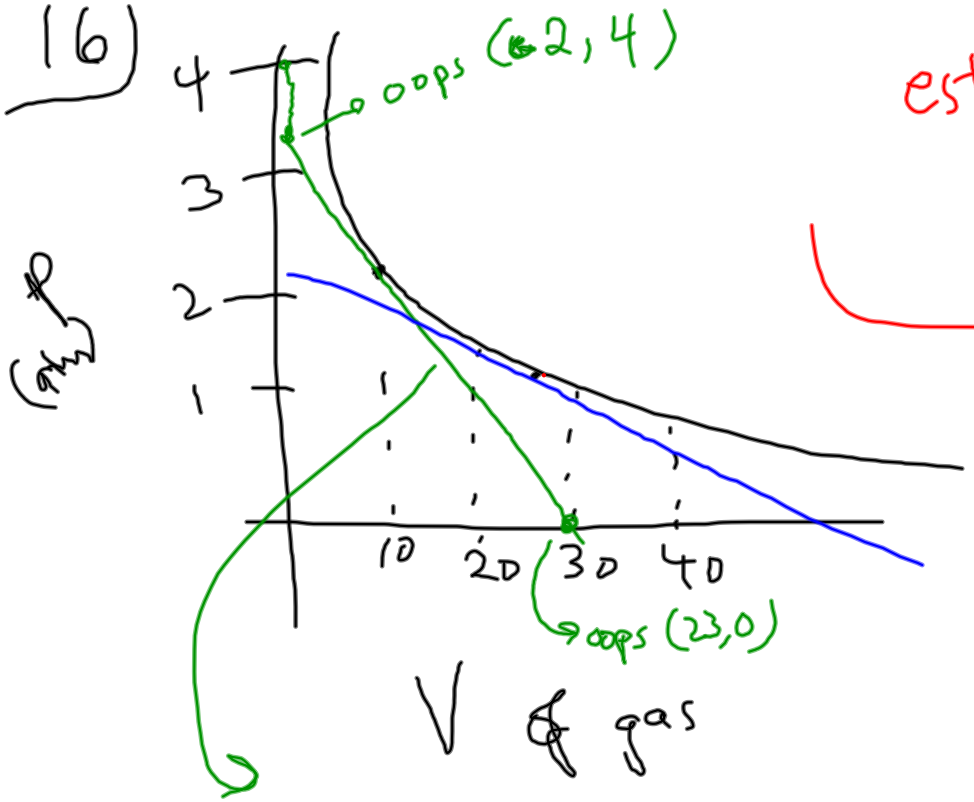
but [because $\sqrt{x_0}$] $x_0 > 0$

$\Rightarrow -\frac{1}{2x_0\sqrt{x_0}}$

b) inst rate @ $x_0 = 4$

$= \frac{-1}{2(4)\sqrt{4}} = \frac{-1}{16}$

16)



estimate inst rate of chg by slope of tangent.

$$m = \frac{0-4}{23-2} = \frac{-4}{21} \approx -.19$$